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Researches on the Elastic Properties and the Plastic Extension of Metals

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IV. *Researches on the Elastic Properties and the Plastic Extension of Metals.*

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§ 1. *Preliminary.*

On March 7, 1912, I described an instrument which gives photographically a load-extension diagram of a metal test piece during the process of stretching it to fracture.

On February 13, 1913, I described further experiments with the instrument.* A diagram was shown which was taken from a test piece broken in ten seconds. It is safe to say that up to that time no apparatus existed which would give a complete record of the load-extension relation during such a quick break.

I have since that time arranged the apparatus to record at even a quicker rate. Fig. 1 shows the record of a break done in 2·15 seconds. The test piece measured

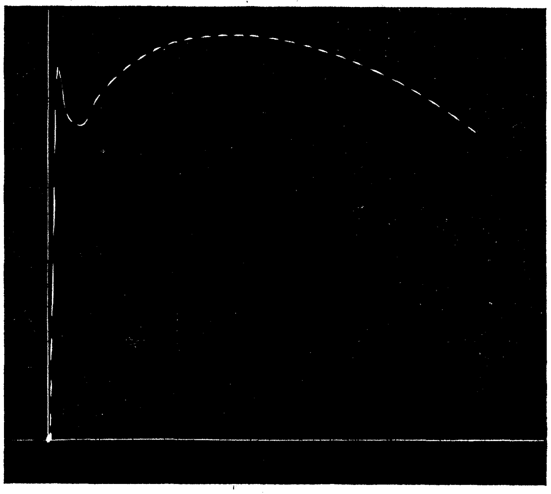

Fig. 1 (mild steel).

one inch between the shoulders. The line of the record is in dashes. These dashes fix the time scale of the diagram. Centre to centre of a pair of dashes corresponds to

* Additional results are given in 'Transactions of the Institute of Naval Architects,' March 29, 1912, 'Institute of Metals,' May Lecture, 1917, vol XVIII., No. 2.

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$\frac{1}{14}$ of a second. This time calibration is obtained by placing an interrupter in the lamp circuit. Referring to the diagram it will be seen that the yield load was reached in about $\frac{7}{14}$ seconds.

In my 1913 paper I included a diagram taken with an instrument which multiplied the extension of the gauge length 150 times so that the elastic part of the curve appeared on a scale which enabled its shape to be studied and which enabled the limit of proportionality to be identified when such a limit existed.

In my method of taking these diagrams the test piece is stretched without pause in the loading and the spot of light follows without break of continuity every phase of the relation between load and extension. Sudden slips of the crystals are duly recorded.

In the usual method the load is applied in steps, pausing at each step to observe the extension, so that the piece gets a rest under steady load during the time occupied in making the observation of extension. The load-extension curve is thus defined by a definite number of points only and peculiarities of form between these points are missed.

I have from time to time continued these elastic researches, and the following paper records some of the results obtained with what I call the *Optical Recorder of Load and Elastic Extension.*

§ 2. *The Test Piece.*

In these researches the gauge length is defined by flanges turned on the test piece itself. The ends of the arms of the extensometer rest on these flanges. The dimensions of the standard form of test piece used in these researches are shown in fig. 2. A shorter gauge length was used for the more ductile metals, but all experiments on the iron and steels were done on a 5-inch gauge length.

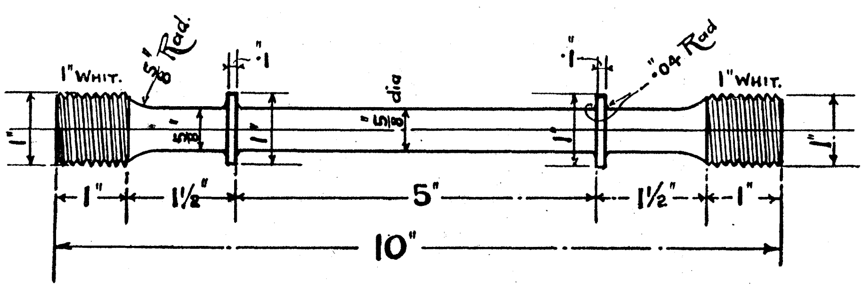

Fig. 2.

I was lead to adopt this form by the many difficulties encountered when pointed screws have to be driven into the test piece to define the gauge length.

These screws cannot be driven properly into hard material like the alloy steels which have to be tested nowadays, and in soft material like copper the primitive centre dot in which the points rest, elongates as stretching proceeds and the points slip.

The modulus of elasticity, E, found from flanged test pieces agrees with the values found from plain bars. The flanges therefore have negligible influence on the elastic extension of the gauge length. They restrict the plastic extension slightly. In mild steel the total extension is about 3 per cent. less when found from a flanged test piece than it would be if found from a plain bar.

Dr. COKER has kindly examined the distribution of stress produced by a flange in a xylonite test piece made to the dimensions of fig. 2. When the xylonite test piece is stretched the colours show that there is no stress in the flange itself and there is a slight but symmetrical modification of the stress distribution at its root. This means that as stretching proceeds the flange is not distorted, and therefore the distance between the flanges is a true measure of the extension of the primitive gauge length which they define.

§ 3. *The Elasticity of Materials and a Typical Load Elastic Extension Diagram of Mild Steel.*

The elasticity of a material means in a general sense its power of returning to its primitive form after loading has been applied and removed.

The recovery may be partial or complete.

The power of complete recovery is lost when the stress produced by the loading has once passed beyond a certain limiting value peculiar to the material.

Below this limiting stress the extension of a steel test piece is proportional to the load.

Above this limiting stress the extension increases at a greater rate than the load.

The limit is therefore called THE LIMIT OF PROPORTIONALITY.

The power of recovery may thus be distinguished into the power of complete recovery possessed and retained only so long as the stress in the steel has never once exceeded the limit of proportionality: and the power of partial recovery peculiar to the state into which the metal passes directly it has once been loaded beyond the limit of proportionality.

Provided that the material has never been loaded beyond its limit of proportionality the material may be said to be in a state of *perfect elasticity*, because it possesses the power of complete recovery of form after removal of load; alternatively it may be said to be in a state of *proportional elasticity* because its extension is found to be proportional to the load.

The one term includes the other. If it is found to extend proportionally to the load its recovery is perfect after removal of load.

No metal is, however, quite perfect in its recovery, but the term perfect used in the sense defined above is convenient and substantially expresses the experimental results.

A diagram recording the elastic extension of mild steel is seen in fig. 3.

This steel contains 0·156 per cent. of carbon. The extension scale of the diagram is defined by the distance between the two vertical lines seen in the diagram. This distance represents an extension of 0·01 inch.

Proportionality between load and extension ceases at about 4·5 tons corresponding to a stress of 14·67 tons per sq. inch. Yield occurs at 6·7 tons which corresponds to

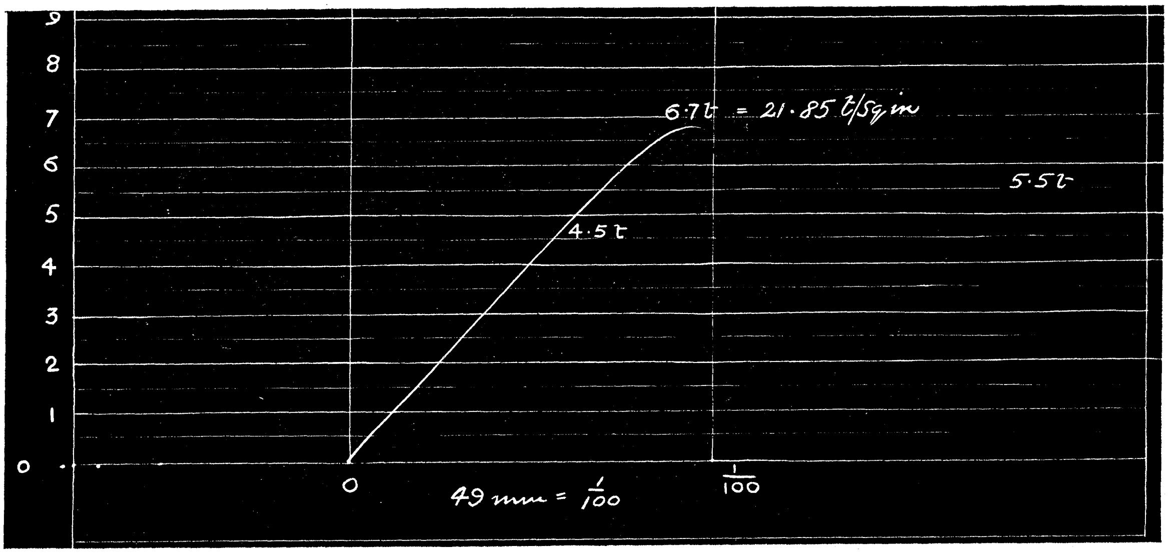

Fig. 3 (mild steel).

21·85 tons per sq. inch. The load drops away from yield to about 5·5 tons giving a stress of 18 tons per sq. inch. These stress are reckoned on the original area of the cross-section of the test piece.

The slope of the line from the origin to the limit of proportionality defines E, the modulus of elasticity. From the diagram its value is 13,300 tons per sq. inch.

§ 4. *Restoration of Perfect Elasticity after Overstrain.*

The term "OVERSTRAIN" means that a metal has been loaded beyond its limit of proportionality.

If the load is removed after a test piece has been strained beyond the limit of proportionality and then the piece is immediately re-tested, the record shows a curved line.

It has no range of proportionality and no modulus of elasticity which can be identified with E.

The material still possesses elasticity because it shrinks as the load is removed, but the elasticity is imperfect in the sense that change of length is no longer proportional to change of load.

But, if the metal is iron or mild steel, proportional elasticity is slowly recovered with time; and this change from unproportional to proportional elasticity or from imperfect to perfect elasticity is accelerated by boiling.

In fact overstrained iron or mild steel is restored to its perfect or proportional elastic state with remarkable rapidity by mere boiling. This point has been established by Sir ALFRED EWING.*

I have found, however, that overstrained high carbon steels and the alloy steels do not recover proportional elasticity either by resting or by boiling.

The elastic line of a 3 per cent. nickel steel is seen in fig. 4. It is lettered A.

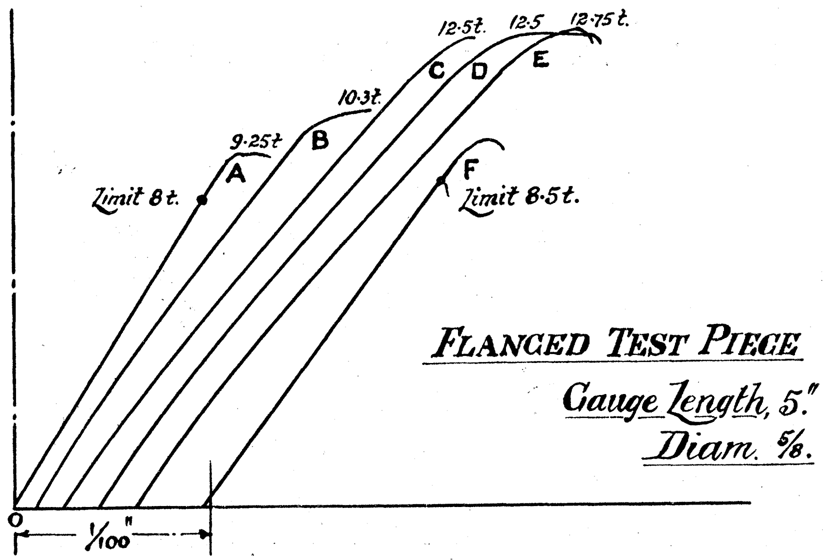

Fig. 4 (nickel steel).

The limit of proportionality is reached at 8 tons, 26 tons per sq. inch, and the yield at 9·25 tons, 30 tons per sq. inch.

The piece was stretched 2 per cent. and then the diagram, line B, was taken. Proportional Elasticity has disappeared.

Curve C is the record after a 6 per cent. stretch.

Curve D is a repetition test after turning the bar to a slightly reduced diameter. The interval of time between C and D is 24 hours. No restoration of elasticity has taken place. It has been established by other experiments that a lapse of many months has no effect in restoring the proportional elasticity.

The piece was then boiled for 1 hour, and curve E shows that elasticity has not been restored.

Finally, the piece was heated to 550° C. in a muffle furnace for about half an hour and was then allowed to cool down with the furnace. Line F, taken immediately after this treatment, shows perfect recovery of proportional elasticity and a slight raising of the limit and the yield point.

I have confirmed these results by other experiments on nickel steel test pieces and on high carbon steel test pieces.

* 'Phil. Trans. Roy. Soc.,' 1899.

§ 5. *Looping after the Elastic Limit of Proportionality has been Passed.*

The recorder is fitted with a microscope so that the process of stretching can be watched as the experiment proceeds, and the loading, which is produced by hydraulic pressure, stopped at any moment. This arrangement enables interesting records to be taken, because after the test piece has been stretched an assigned amount, the load can be let off and then immediately re-applied, so that stretching continues through a second interval and so on.

Such a record is seen in fig. 5. The material is nickel steel. It will be seen that

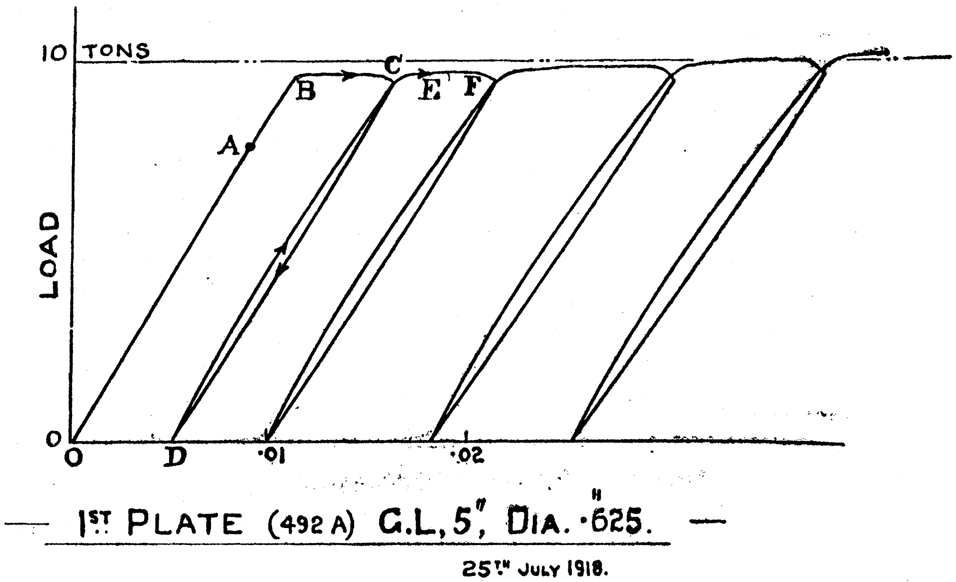

Fig. 5 (nickel steel).

the removal and the re-application of the load compels the spot of light to trace a loop. The area of the loop represents the internal work done during the process.

Following the path of the spot it starts from the origin O and describes the elastic line OA, passes the limit of proportionality at A, and then curves away to the yield point B, and on to C. At C the loading is stopped, the hydraulic pressure is relieved by opening the exhaust valve, and the spot travels down the curved path CD as the load falls to zero. The exhaust valve is then closed and the pressure valve is opened and the process is repeated through the path EF and so on.

When the steel test piece has been stretched beyond its limit of proportionality, for example to C, fig. 5, the total extension is made up of two parts, namely :—

(1) the proportional elastic extension up to the limit of elasticity, for example up to A, fig. 5;

(2) the plastic extension after the limit has been passed.

When the load is removed the steel shrinks unproportionally, for example from C to D, by an amount approximately equal to its previous proportional elastic extension and then stops at a dimension greater than its primitive dimension by an amount approximately equal to its plastic extension.

The increase of size measured after the removal of the load is called the Permanent Set. OD is the permanent set produced by the first stretching of the test piece to C. From the diagram the permanent set measures 0·0052 inch and the unproportional elastic recovery measures 0·011 inch.

When the load is re-applied the spot of light moves from D to E along a curved path. The extension is no longer proportional to the load. The metal is in a different elastic state. Stretching beyond the limit of proportionality has robbed the metal of its power of proportional extension and of perfect recovery after removal of load. It may be said to be in a state of *imperfect elasticity*, or alternatively it may be described as in a state of *unproportional elasticity*.

The imperfect state is disclosed by the loop formed by the removal and re-application of the load.

The diagram shows four loops. Each loop is slightly larger than the loop preceding it.

The four loops shown were all recorded on a half plate inserted in the camera. A succession of plates was taken and the last plate is shown in fig. 6.

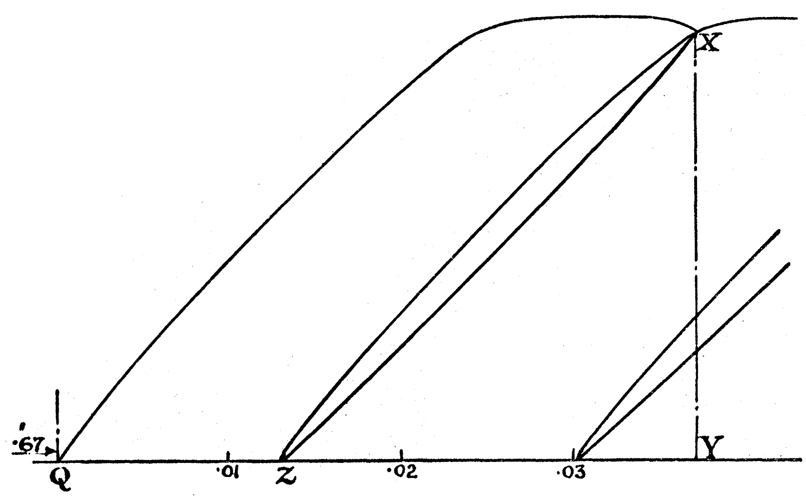

Fig. 6 (overstrained nickel steel).

This last plate shows that unproportional elastic shrinkage occurs right up to the load at which local contraction begins. The last loop is just seen on the plate.

The last line seen curving up from the origin Q is the typical curve of overstrained material. The primitive gauge length of 5 inches had been stretched to 5·67 inches before the last plate, fig. 6, was taken.

The permanent set QZ measured from the new origin Q is 0·0127 inch. The total permanent set is thus 0·67 + 0·0127 inch = 0·6827 inch. The unproportional elastic recovery is ZY = 0·0244 inch.

§ 6. *Looping a General Property of Metals.*

Records of looped diagrams are shown in the following figures. The load scale is varied to bring out the shape of the loops.

The extension scale is substantially, $1\frac{1}{2}$ inches measured horizontally on the diagram represents $\frac{1}{100}$ inch extension of the gauge length. The diagrams are reproduced as taken, the object being to compare the elastic line and the loop formations.

Staffordshire Iron.—Fig. 7. The limit of proportionality is reached at 3·5 tons;

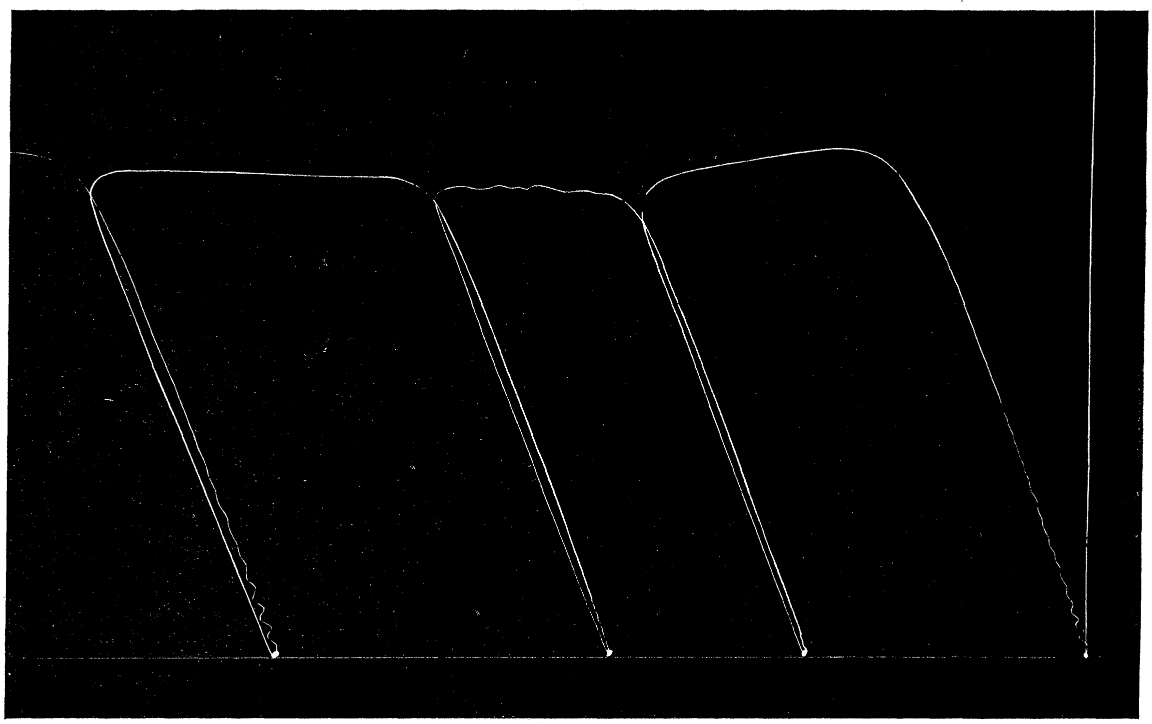

Fig. 7 (iron).

11·4 tons per sq. inch. The yield is reached at 5·1 tons; 16 tons per sq. inch. The load then drops to about 4·7 tons. The loop area is not large, but the area increases progressively.

Steel.—Carbon 0·8 per cent., fig. 8. The limit of proportionality is reached at about 8 tons; the yield at 8·85 tons. There is a slight drop at the yield load. The area of the first loop is many times larger than the area of the first loop of the previous record, and this area increases rapidly in the succession of loops.

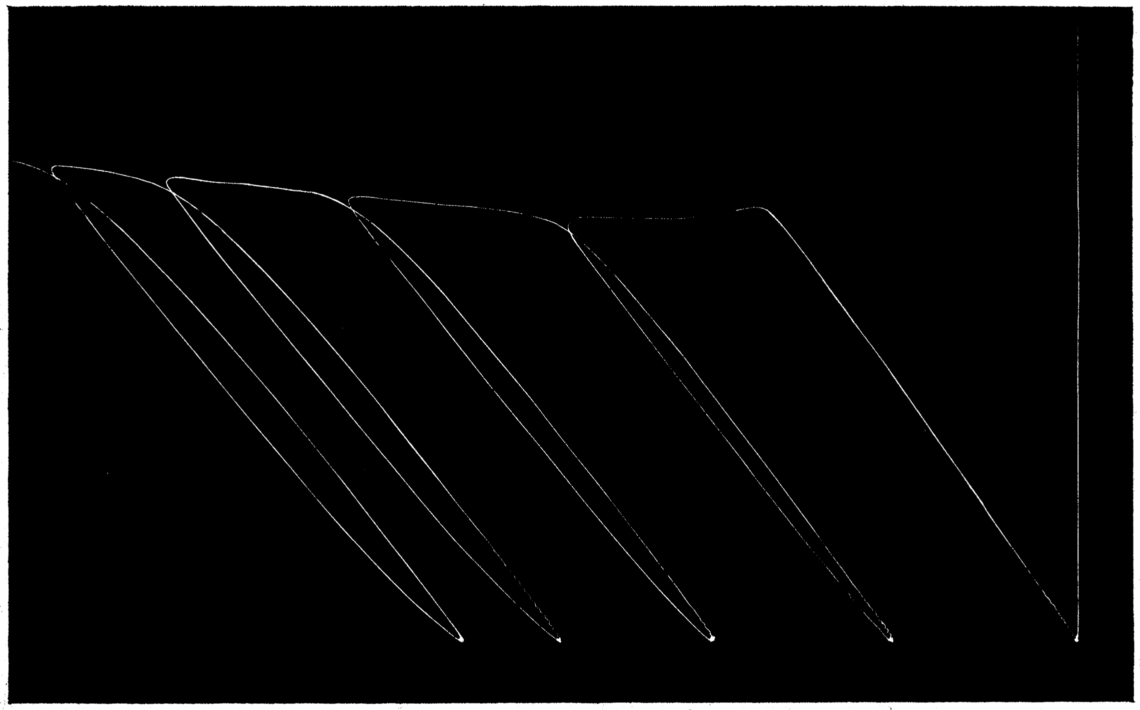

Fig. 8 (0·8 carbon steel).

Steel.—Carbon 0·8 per cent., fig. 9. This diagram is introduced because it is taken from a test piece cut from a bar delivered from the works as steel of the same kind and quality as that from which the previous diagram was taken.

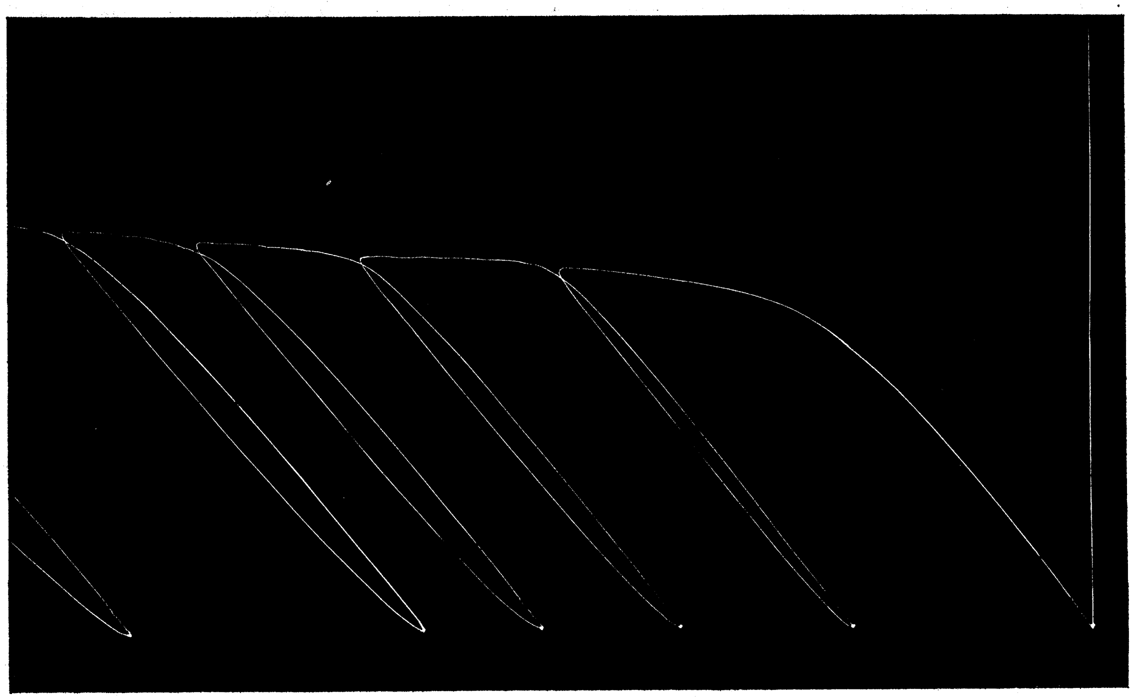

Fig. 9 (0·8 carbon steel).

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The limit of proportionality occurs now at about 3 tons and there is no definite yield point. The loop areas and their rates of increase are about equal in the two plates.

The explanation of the difference in quality shown by comparing the two diagrams may be found in the fact that the test piece of fig. 8 was cut from a bar delivered before the war. The test piece of fig. 9 was cut from a bar delivered towards the end of the war. There has clearly been some change in the manufacturing process.

Nickel Chrome Steel.—Fig. 10. The ultimate strength of this steel found from a

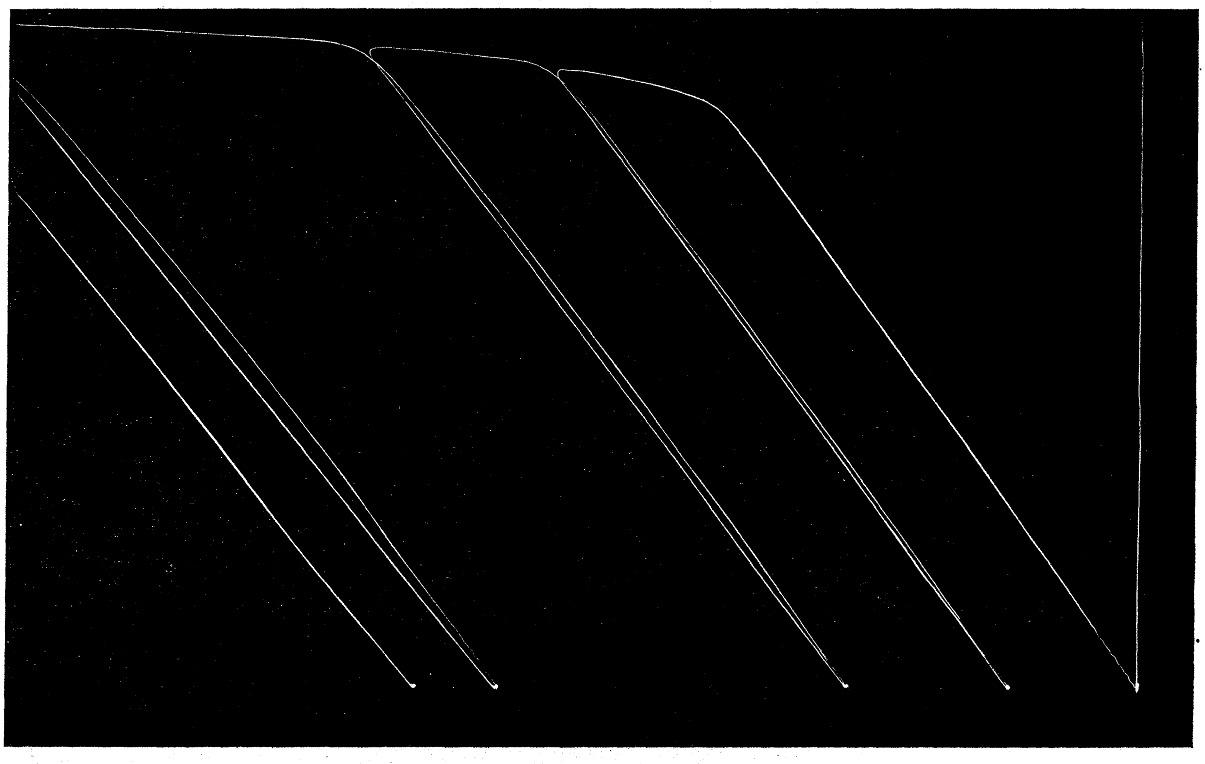

Fig. 10 (nickel chrome steel).

bar 1 inch diameter is 54 tons per sq. inch, with an extension of 14 per cent. on 8 inches and 55 per cent. reduction of area. The limit of proportionality is at a load of 10 tons on the standard test piece 0·625 inch diameter, corresponding to 32·5 tons per sq. inch. Yield sets in at 11 tons, that is, 36 tons per sq. inch.

The first loop of the diagram is small, but the area increases rapidly, as will be seen from the three loops visible in the record.

Nickel Steel.—Carbon 0·33 per cent., Ni 3·52 per cent.—Fig. 11. The ultimate strength of this material is about 48 tons per sq. inch, with an elongation of 20 per cent. on 5 inches and a reduction of area of 47 per cent. Limit of proportionality occurs at about 30 tons per sq. inch and yield at 32 tons per sq. inch. The limit of proportionality here approaches quite near to the yield point.

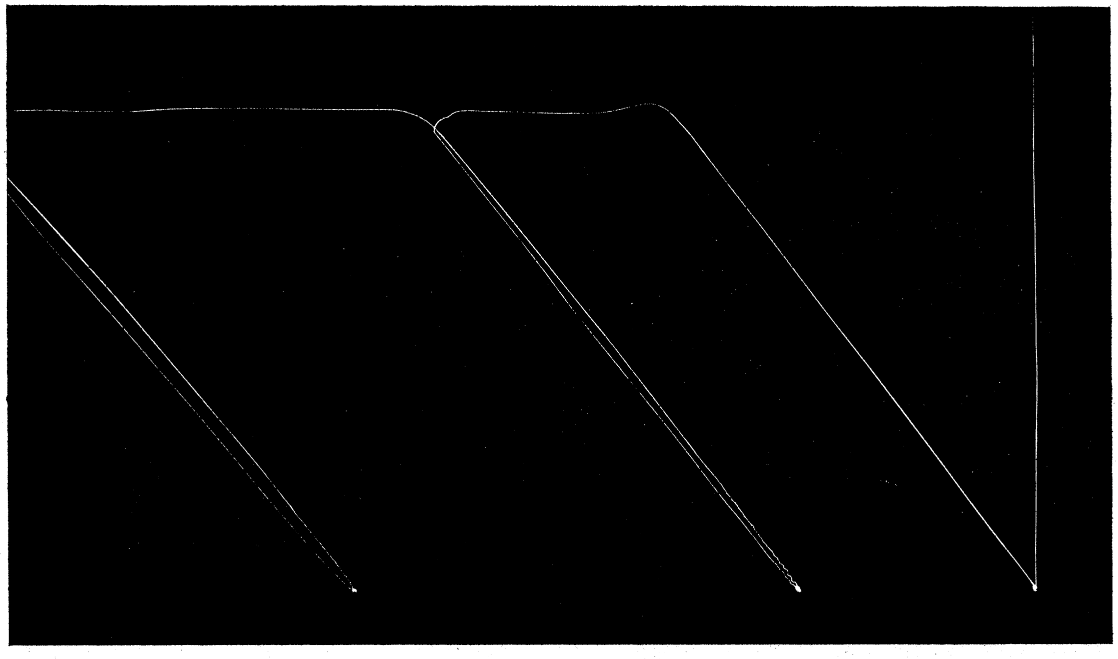

Fig. 11 (nickel steel).

Zinc.—Fig. 12. Diameter of test piece 0·8 inch. Gauge length 5 inches. The test piece was turned from a zinc rod. There is no proportional elastic line. Curvature

Fig. 12 (zinc).

begins at the origin, so that the extension is increasing at a greater rate than the load from the commencement of loading. There is no definite yield point.

The most interesting result to notice is that the test piece goes on shrinking in length after the load has been removed. It is shrinking under the action of its own internal molecular forces because it is entirely free from external load.

The shrinking at no load is indicated by the flat bottom of the loop. A dwell of $1\frac{1}{2}$ minutes was made in the experiment after the removal of the load and before the re-application of the load. All the perceptible shrinking at no load takes place within this time interval.

At the third loop after the load was removed the light was shut off and flashed on at intervals of $\frac{1}{10}$ seconds to get some idea of the rate of shrinking.

Tin.—Fig. 13. Diameter of test piece 0·8 inch. Gauge length 5 inches. This

Fig. 13 (tin).

test piece was turned from a bar of tin. It exhibits properties similar to zinc on a smaller scale. There is shrinking continuing for about 1 minute after the load has been removed, and there is the same absence of a proportional elastic line.

Copper.—Pure and free from arsenic.* Fig. 14. Diameter of test piece 0·8 inch. Gauge length 5 inches. There is no marked limit of proportionality and no yield point in this material. The noteworthy feature of the record is the small rate of increase of loop area. This small rate of increase of loop area is a common

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characteristic of all the copper samples which I have tested. It may be that this rate of increase is identified with the quality of toughness.

Fig. 14 (electrolytic copper).

Copper.—Fig. 15. 99·4 per cent. copper. Arsenic present, and by difference estimated at 0·4 per cent. Diameter of test piece 0·8 inch. Gauge length 5 inches.

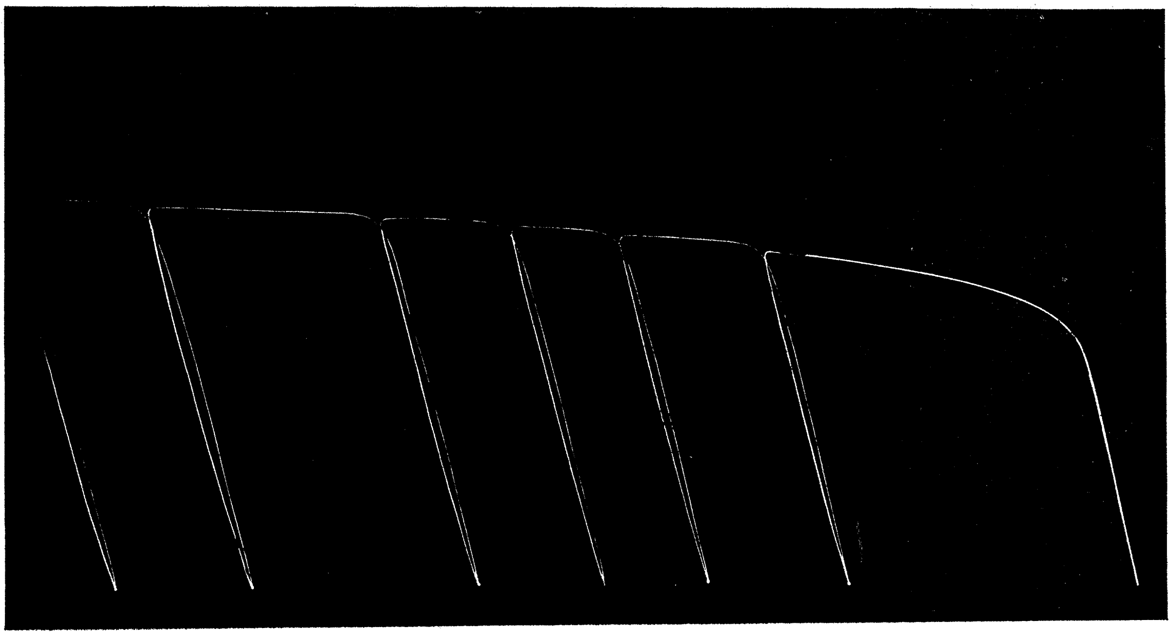

Fig. 15 (arsenical copper).

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The effect of the arsenic is remarkable. It gives to the copper an elastic line with a distinguishable limit of proportionality of 1·4 tons; 2·8 tons per sq. inch.

There is no definite yield point. The loops are small and the rate of increase of loop area is small.

The elastic line from the origin to 1·1 tons is thicker than the continuation of the line. This thickening is brought about by a removal and a re-application of the load. The spot of light travelled three times up and down this piece of the diagram, indicating that the elastic line, within the limits of this load, is permanent.

Brass.—Fig. 16. Composition 60 per cent. copper, 40 per cent. zinc, with traces

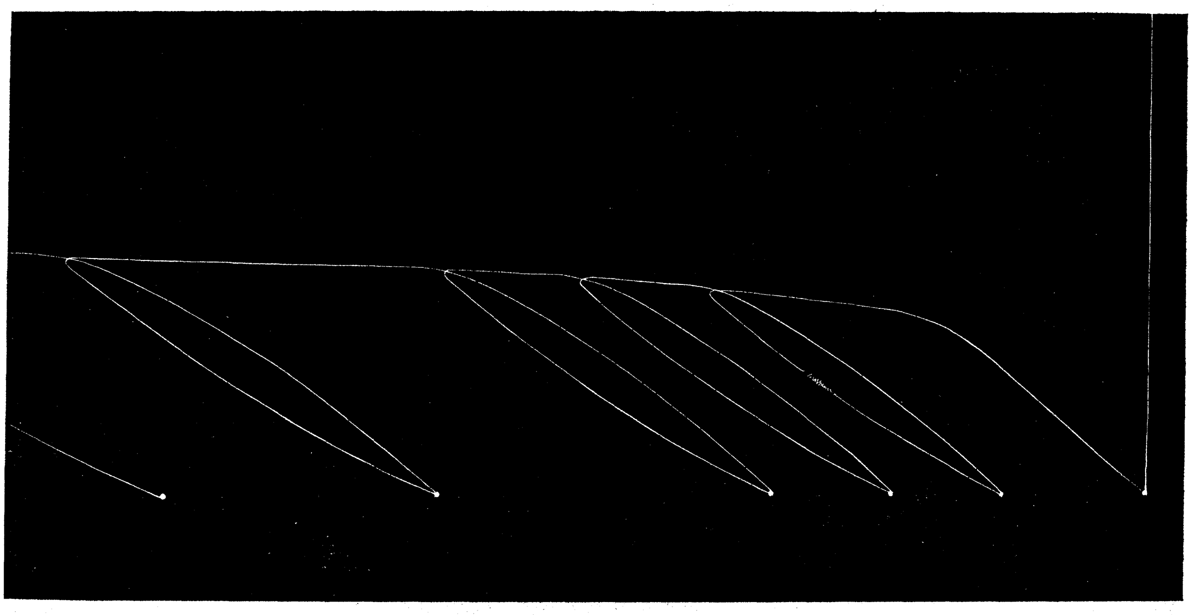

Fig. 16 (brass).

of tin and other impurities. The ultimate strength of the material is 32·6 tons per sq. inch. There is a marked limit of proportionality at $2\frac{1}{4}$ tons; 7·33 tons per sq. inch. Yield follows gradually. There is no contraction after the load has been removed, although the material contains so much zinc.

Phosphor Bronze.—Fig. 17. The curve in this diagram shows a limit of proportionality at about 2 tons; 6·5 tons per sq. inch; but it is difficult to locate the exact spot at which the line begins to curve away from the primitive straight element.

Aluminium Alloy.—Fig. 18. Diameter of test piece 0·625 inch. Gauge length 5 inches. This diagram is remarkable in that the removal and the re-application of the load in the plastic state shows no looping and therefore no hysteresis loss which can be calculated from the loop area. It appears as though the metal continually anneals itself at ordinary temperatures as plastic stretching proceeds. The material appears to be elastic up to a load of $4\frac{1}{2}$ tons. The thick line indicates the removal and the re-application of load before the metal begins to yield plastically.

Fig. 17 (gun metal).

Fig. 18 (aluminium alloy).

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§ 7. *Loop Area and Permanent Set.*

The loop area increases in size as the stretching proceeds and the rate of increase differs in different materials.

The question now arises: does the increase in area follow a regular law? The answer is given by the curves on Sheet 1 (folding diagram).

The co-ordinates on Sheet 1 are loop area and permanent set. Curve 1 shows the results obtained from a test piece of 0·8 inch carbon steel, $\frac{5}{8}$ inch diameter, with a 5 inch gauge length. The slope of this curve shows the rate of increase of loop area as stretching is continued. The curve ends when local contraction begins. Similar curves are given on Sheet 1 for nickel steel, mild steel, and for iron.

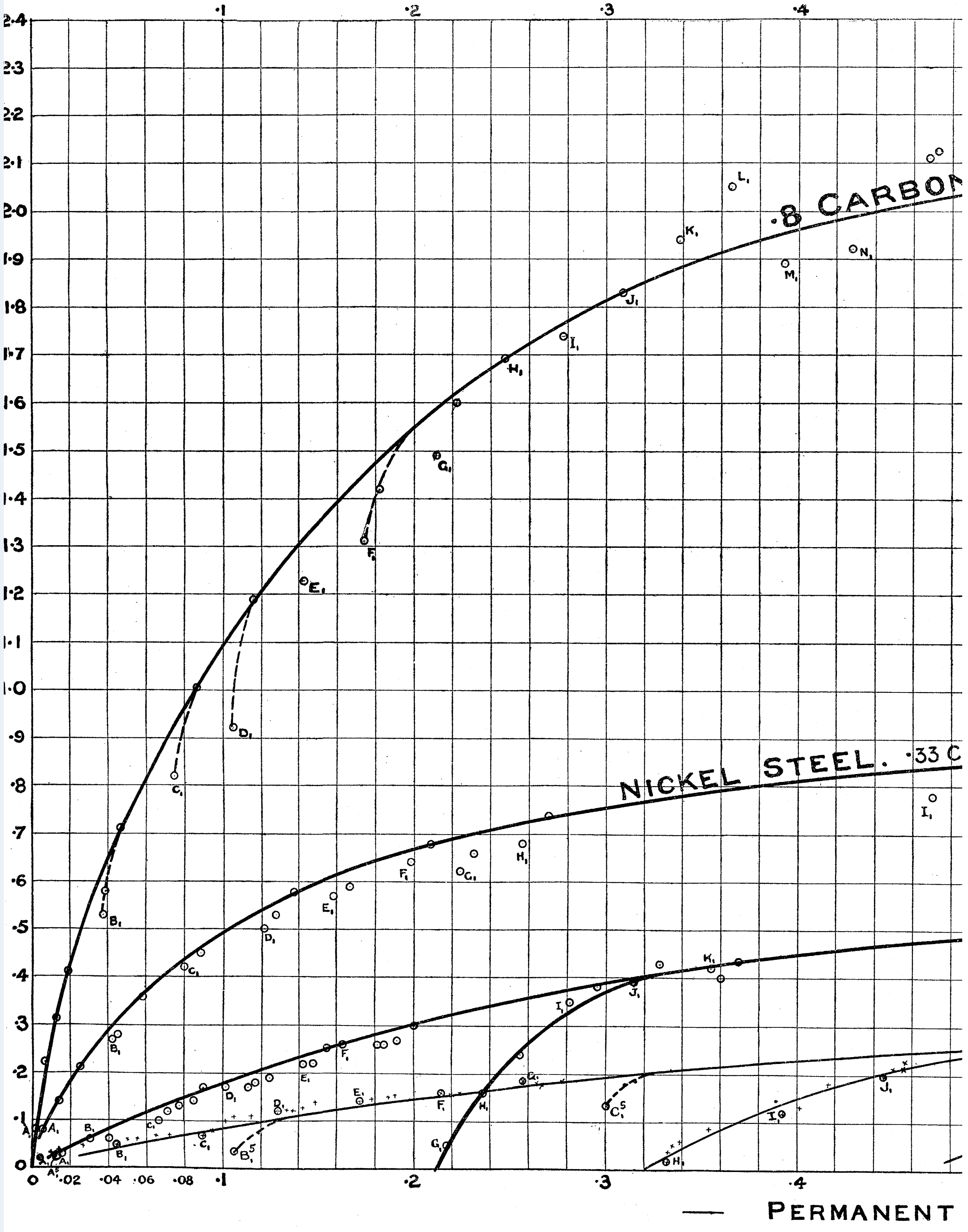
The rate of increase depends upon the time interval between the drawing of the loops and upon the kind of material. In irons and mild steels the influence of time is profound. In the alloy steels tested and in high carbon steels the influence of time is small.

When the stretching of iron or mild steel is resumed after a rest, the loop area, at first small, increases rapidly towards the area the loops would have had if stretching and looping had been continued without resting. Anticipating the detailed description of the curves on Sheet 1, this point may be illustrated by curve 3, same sheet. Plates A_1, B_1, C_1, D_1, E_1, and F_1 were taken consecutively, there being no more time interval between the plates than the few seconds required to change the plates. After the mild steel test piece had been stretched to 0·2 inch it was taken out of the machine and laid aside for 15 days. Plate G_1, the first plate taken after the rest, and plates H_1, I_1, J_1 furnish loops of rapidly increasing area until the area is reached on plate K_1, corresponding to continuous stretching without rest intervals.

§ 8. *Loop Area and Permanent Set Curve. Curve* 1 *Sheet* 1 (0·8 *per cent. Carbon Steel*).

The detailed consideration of this curve will show how all the curves of the diagram on Sheet 1 have been derived. The capital letters along the top of the procession of loops seen in fig. 19 refer to the sequence of negatives recording the loops taken from a standard test piece of 0·8 inch carbon steel.

Plate A gives the record of the first application of the load to the test piece and its immediate removal and re-application four times. The plate therefore shows the elastic line and the first four loops. A *scale* is placed under the loops so that the *permanent set* of the primitive 5-inch gauge length can be read at any point in the procession of loops. For example, the permanent set at the end of the looping operations recorded on the sequence of plates A, B, C, D, is the distance $0k = 0·137$ inch.



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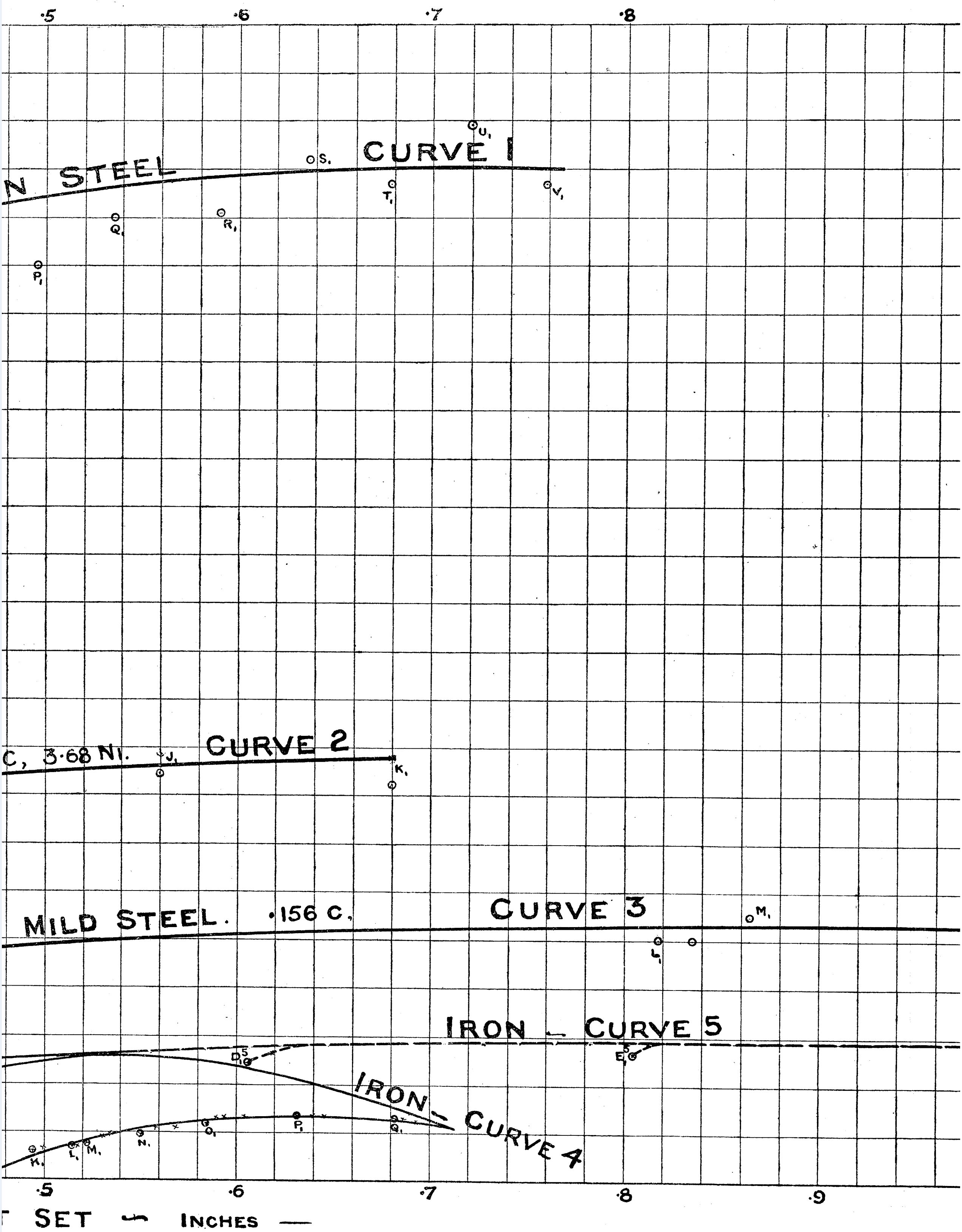
Each small circle denotes a loop, and the letter w



— LOOP AREA PLOTTED

e letter written against some of them identifies the first loop on the plate corresponding with the letter. The subscript 1 r

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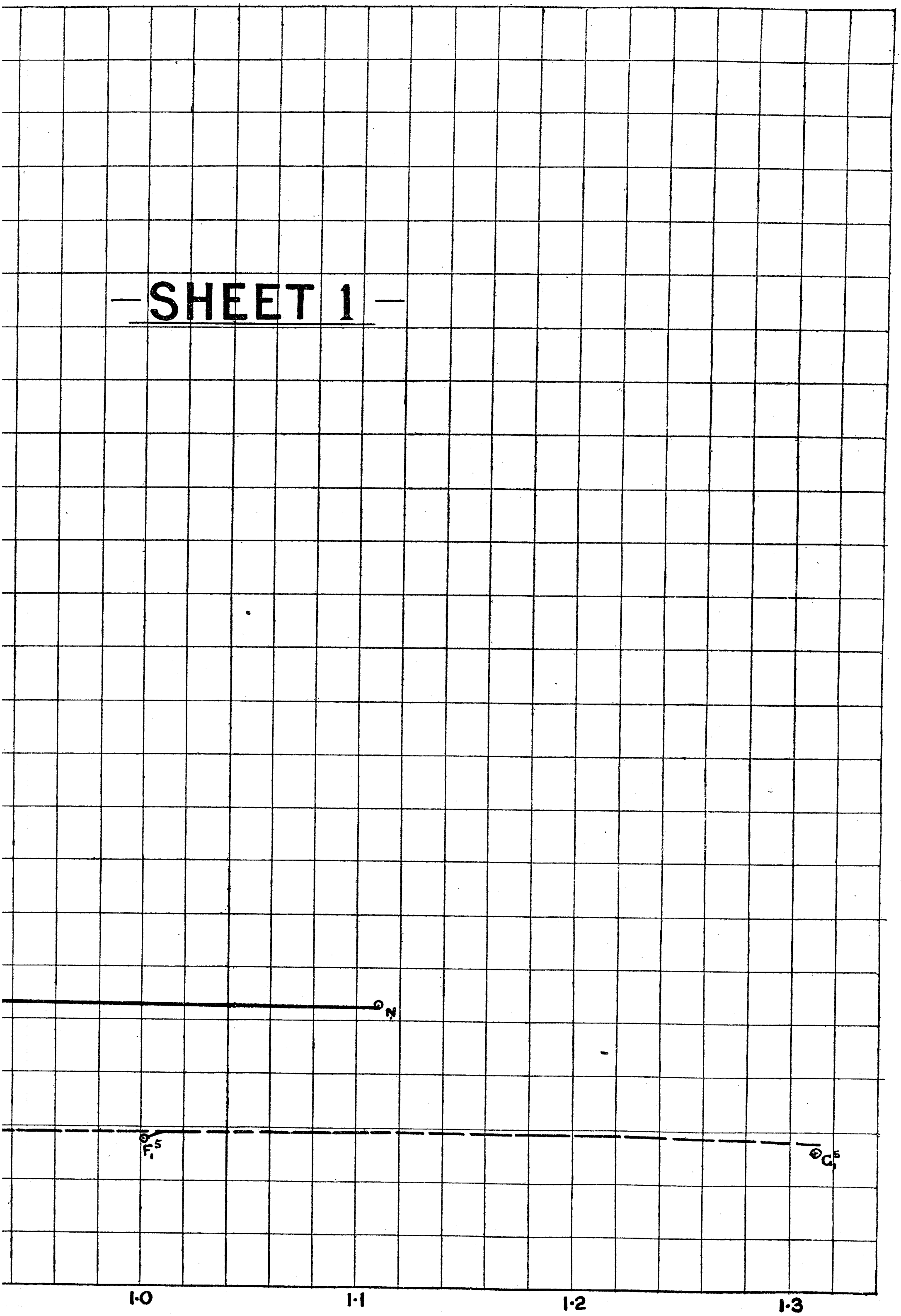

TED AGAINST PERMANENT SET

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Each small circle denotes a loop, and the letter w

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—— LOOP AREA PLOTTED

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TED AGAINST PERMANENT SET

script 1 means the 1st loop.

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The procession is formed by setting the plates in due sequence and placing the origin of the record on each plate at the point on the extension scale corresponding to the permanent set measured from the test piece itself. For example, direct measurement of the gauge length after taking plate D shows that the permanent set is 0·137 inch. The origin of the record on plate E is then located at 0·137 inch on the scale.

The procession of loops seen in fig. 19 is reproduced on a small scale in order to present to the eye the complete record on a reasonably sized sheet.

Selected loops from the procession are shown full size in figs. 20 to 23. The elastic line and the first and second loops are seen in fig. 20. The area of the first loop represents an energy loss of 0·42 ft. lbs., and of the second loop 1·15 ft. lbs. The corresponding permanent sets are 0·002 inch and 0·007 inch.

The areas through the sequence of plates A, B, C, D, E, F increase gradually. The last loop of this sequence is seen in fig. 21 and it represents an energy loss of 7·42 ft. lbs. The time occupied in taking these six plates was 23 minutes.

After taking plate F the experiment was stopped. The test piece was removed from the machine and laid aside. After six days' rest it was put back into the machine and looping continued.

The first line after the rest and the first loop are seen in fig. 22. The first line shows no elastic recovery and the six days' rest has had no perceptible influence on the loop area, which represents 7·78 ft. lbs. The area is what it would have been if there had been no interruption of the experiment for a period of rest. The sequence of plates G to V was taken in 1 hour 20 minutes. The record on the last plate is seen full size in fig. 23. Stretching was stopped because local contraction had set in.

The areas of the loops in the sequence are plotted against permanent set in curve 1, Sheet 1. Each small circle denotes a loop, and the letter written against some of them identifies the first loop on the plate corresponding with the letter.

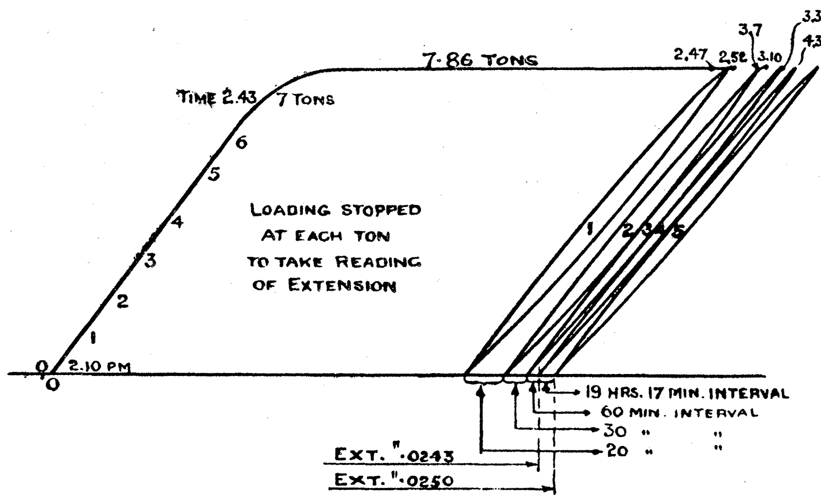
It will be noticed on Sheet 1 that the curves joining the loop areas on any one plate do not merge into one another to form a continuous curve. The time interval required to change the plate and to resume loading seems to be occupied by the material in some inner process which tends to slightly reduce the area of the next loop taken. But whatever the inner process may be, it practically exhausts itself in a few moments and produces only slight effect on the next loop area. No further change takes place after a rest of six days, and the inner process, whatever it is, has no influence in restoring the material to a state of perfect elasticity after the overstrain.

A curve sketched through the group of loops on each plate is continuous and clearly shows that the area of the loops tends to a maximum. The maximum value in this experiment represents an energy loss of 11·48 ft. lbs. per loop. This loss corresponds to 7·51 ft. lbs. per loop per cubic inch of material in the primitive gauge length.

Processions of loops were similarly taken from a nickel steel test piece, from a mild steel test piece, and from an iron test piece.

§ 9. *Looping under Constant Load.*

The diagram in fig. 24 shows the effect of looping under constant load.



LOOP	AREAS	
1	0·25 sq. in.	PL. 562 A
2	0·18 " "	
3	0·155 " "	
4	0·155 " "	
5	0·17 " "	PL. 562 B

— LOOPING UNDER CONSTANT LOAD — FIG. 24

— 0·8 CARBON STEEL —

The test piece was placed in a Buckton testing machine and loaded gradually until yield began at 7·86 tons, and then loading was stopped. The extension was allowed to proceed under this load for 4 minutes. Then the load was removed and re-applied and the spot of light traced out loop 1.

Extension continued slowly under the load, still maintained at 7·86 tons, and loops 2, 3 and 4 were taken at intervals of 20, 30 and 60 minutes respectively.

The next interval was 17 hours 20 minutes, during which time the gauge length extended $\frac{1}{1000}$ inch approximately. Loop 5 was then taken. Comparing these loops it will be seen that there is no recovery of proportional elasticity although the piece was allowed to stretch under the yield load of 7·86 tons for about 20 hours.

This shows that if the yield load is kept on until the test piece has stopped extending, a process which may take a long time, at the end of the experiment the test piece will not have gained proportional elasticity. It is still in a state of imperfect, or unproportional elasticity.

§ 10. *The Practical Utility of the Load Elastic Extension Looped Diagram.*

A diagram showing the elastic line and a few loops is of great practical utility in industrial applications. The data immediately measurable from the diagram are :—

(1) the load at the limit of proportionality ;

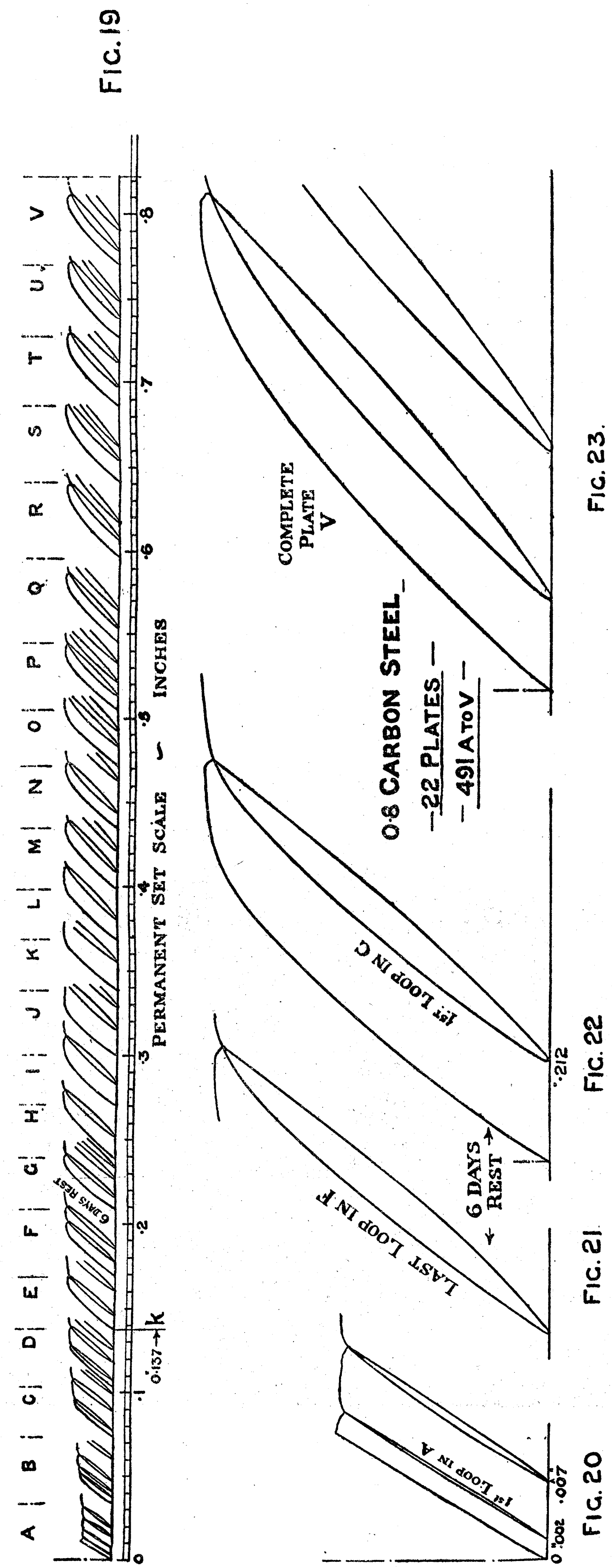


FIG. 19.

FIG. 20. FIG. 21. FIG. 22. FIG. 23.

(2) the yield load;
(3) the value of E: this is given by the slope of the elastic line;
(4) the work lost per loop;
(5) the rate of increase of the work lost per loop.

The diagram from *normal material* satisfying known conditions of composition and manufacture may be used as a diagram of comparison. The form of the curve, the loop area and the rate of increase are sensitive to changes in the kind of material and to changes in the inner state of materials.

The diagram is specially useful in showing the load at the limit of proportionality, for this load bears neither a constant relation to the yield point, when there is one, nor to the ultimate load. Consequently factors of safety reckoned against either the yield load or the ultimate load are ambiguous.

This is specially important in gun design. The whole theory rests upon the elastic property of the material, and the theory ceases to apply after the limit of proportionality is passed.

Considerable research in many directions is necessary before a full interpretation can be given to the looped diagrams, and for the present I will reserve further discussion.

§ 11. *Correlation of Diverse Tests by the Load Extension Diagram.*

Load extension diagrams of the kind shown in this and former papers are likely to be useful to the engineer and metallurgist in the correlation of the many different tests now made to ascertain the quality of metals.

For example, fig. 25 was taken from a test piece of material giving a low impact number. Its shape differs markedly from the shape of a normal diagram. It corresponds in fact with the shape of the curve found from overstrained material.

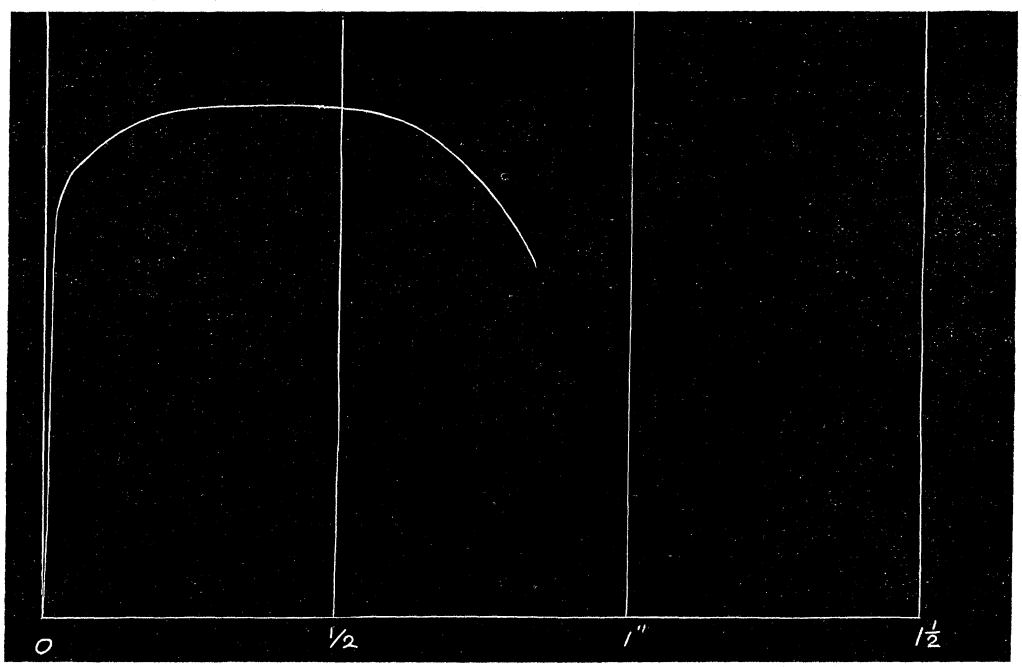


Fig. 25 (overstrained mild steel).

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The inference is that the material is in the overstrained condition. That this inference is correct is shown by fig. 26. A test bar of the same material was annealed by heating

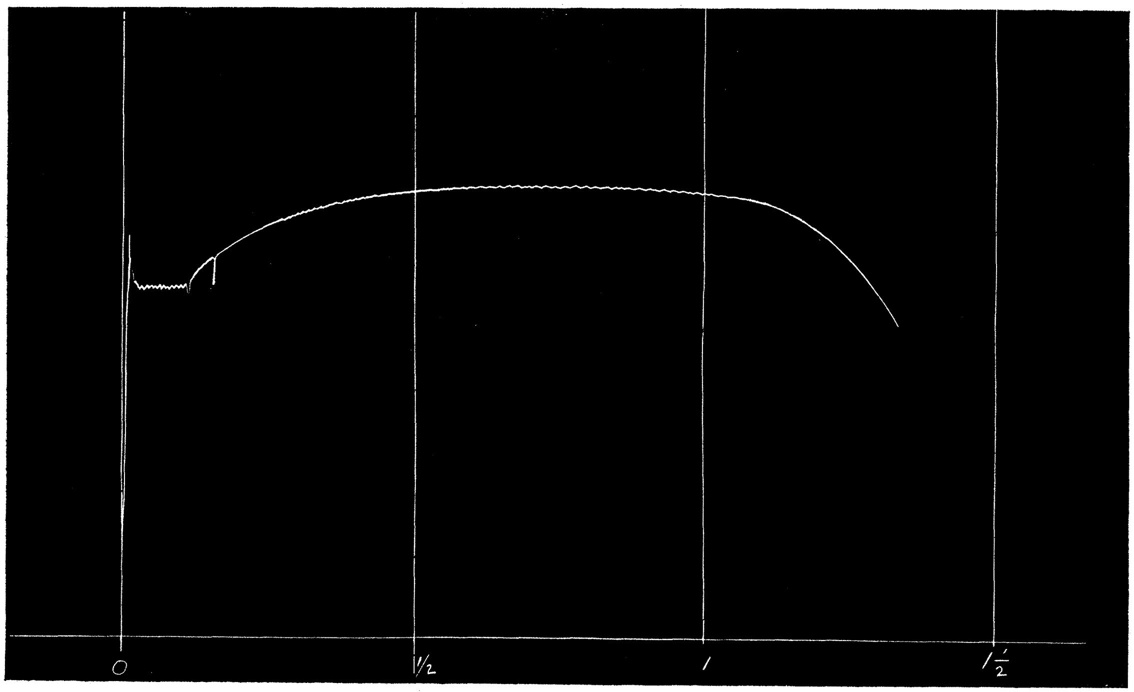

Fig. 26 (normal mild steel).

to 550° C., and then cooling slowly within the furnace, and then it was found that it gave the diagram of fig. 26, which is the normal shape for the class of steel tested.

I have tested many bars of steel rejected on shock test and giving low impact numbers, and I have always found that the shape of the load extension diagram discloses the abnormal state of the metal. Much work of a comparative kind must be done before the result can be widely generalized. The War Committee of the Royal Society did a considerable amount of work in this direction.

The limiting fatigue stress may probably be found by inspection from a load elastic extension diagram.

It is probable that *the limiting range of stress in fatigue has for its positive value the stress equal to the limit of proportionality.*

Referring to the diagram for iron, fig. 7, it will be seen that the limit of proportionality is at about $3\frac{1}{2}$ tons, corresponding to 11·45 tons per sq. inch.

I prepared six test pieces of the material, and Dr. Stanton kindly applied his fatigue test to them at the National Physical Laboratory. He found, after applying alternating loads in the aggregate 24,000,000 times to the eight test pieces, that the approximate limiting range of stress in fatigue was between $\pm 10\frac{1}{2}$ and ± 13 tons per sq. inch. The average is $\pm 11{\cdot}75$ tons per sq. inch.

The agreement between the limit of proportionality shown on the diagram, namely, 11·45 tons per sq. inch, and the fatigue limit found by quite a different test, is remarkably close.

Again, if this result could be generalized, it could be asserted that every load elastic extension diagram shows the positive value of the fatigue limit. The long and tedious experiments with alternating loads would be unnecessary. Such a conclusion requires comparison to be made over a wide range of material. This, therefore, is a promising field of research.

These brief notes of results and inferences show what a wide range of information lies before the engineer and metallurgist if he has before him a pair of diagrams, the one showing the complete load extension curve from zero to fracture, the other a load elastic extension looped diagram on a large extension scale.

The matter incorporated in this paper has been selected from experiments extending over several years. I desire to express my acknowledgments and thanks to Prof. WITCHELL for his help, and in particular for the assistance he gave me in reducing the looped diagram to the curves of Sheet 1.

I also desire to acknowledge the assistance of Mr. ORR for the skill and care with which he has drawn the curves on Sheet 1 and figures 19–23 from the photographic plates.

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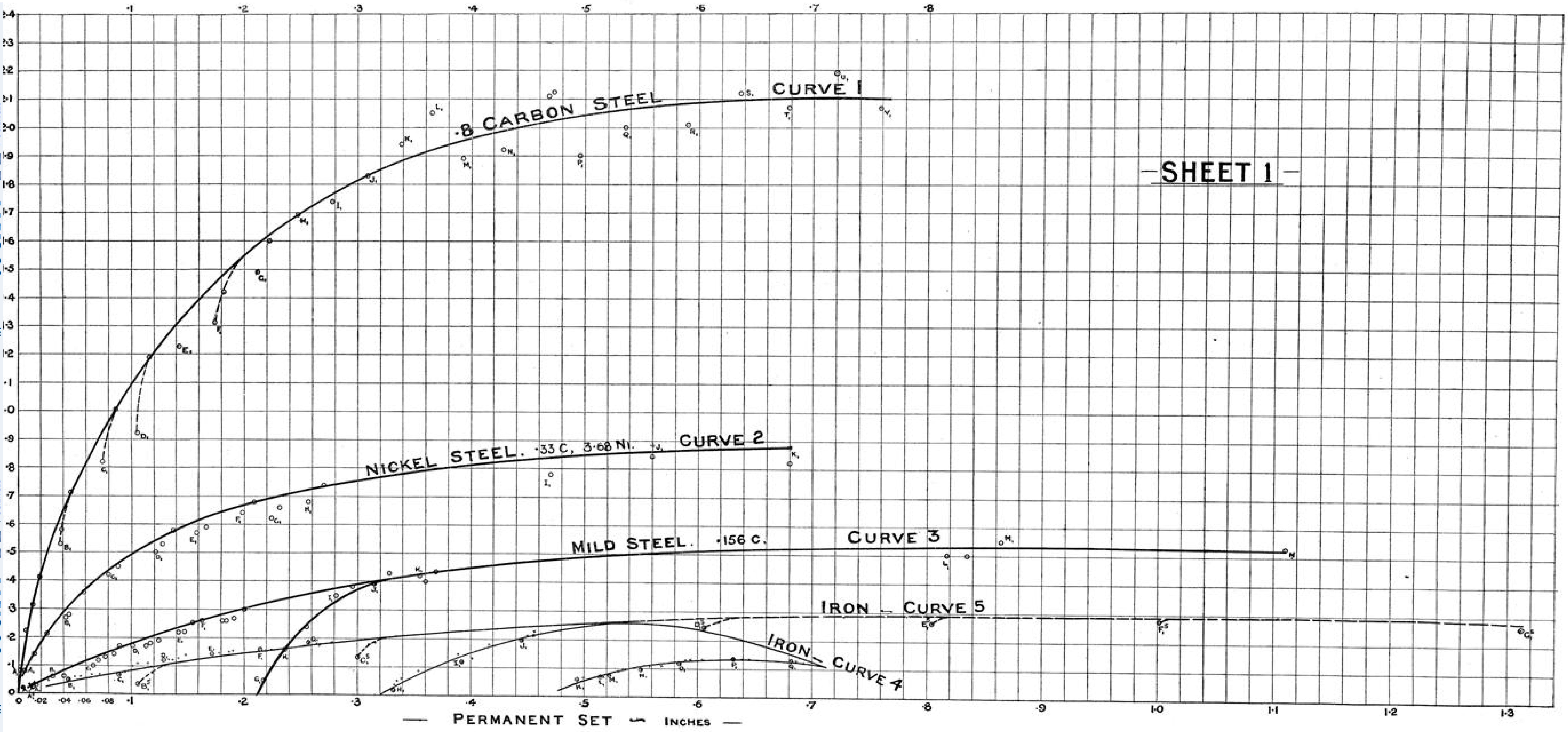

— LOOP AREA PLOTTED AGAINST PERMANENT SET

Each small circle denotes a loop, and the letter written against some of them identifies the first loop on the plate corresponding with the letter. The subscript 1 means the 1st loop.

To face p. 132.

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